Discrete quantum Fourier transform in coupled semiconductor double quantum dot molecules

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In this letter, we present a physical scheme for implementing the discrete quantum Fourier transform in a coupled semiconductor double quantum dot system. The main controlled-R gate operation can be decomposed into many simple and feasible unitary transformations. The current scheme would be a useful step towards the realization of complex quantum algorithms in the quantum dot system.

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The solid-state quantum computation attracts many interests since solid systems are more easily integrated into large quantum networks. Semiconductor quantum dots are considered to be one of the most promising candidates for quantum computation in solid state [1]. Recently, qubits, encoded on the electron-spin singlet state $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ and triplet state $|T\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ of coupled double quantum dot (DQD) molecules, which are being widely researched in theory and experiment [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. It is shown that the coherent time of electron-spin state is comparatively long comparing with the state of charges, and the qubits can be protected from low-frequency noise and can suppress the dominant source of decoherent from hyperfine interaction. Due to their advantages, many schemes based on the two spin states in coupled DQD molecules have been proposed, such as the realization of Bell-state measurement [14] and the generation of cluster states [15], etc.

Inspired by above ideas, we propose a scenario for the implementation of discrete quantum Fourier transform (QFT) via coupled QDQ molecules arranged in line. As we known, only a few physical schemes of QFT have been proposed, one based on cavity quantum electrodynamics (QED) [16] and another based on nuclear magnetic resonance (NMR) [17]. QFT does not speed up the classical tasks of classical Fourier transform (CFT), however, it provides a first step towards the implementation of Shor's factoring and other quantum algorithms, *i.e.*, it is the key ingredient for order-finding problem, factoring problem, counting solution, the solution of hidden subgroup problem and so on. Therefore QFT is still very important in quantum computation. For discrete CFT of N inputs x_j ($j = 0, 1, 2, \cdots, N-1$), the outputs y_k ($k = 0, 1, 2, \cdots, N-1$) can be expressed as

$$y_k \equiv \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi i j k/N}.$$

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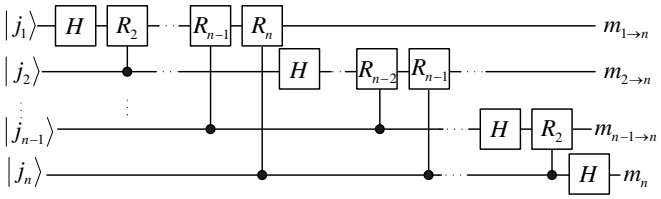

FIG. 1: The effective circuit diagram of discrete QFT [18]. j_l ($l = 1, 2, \cdots, n$) are inputs and $m_{l\to n} = (|0\rangle + e^{2\pi i 0.j_l \cdots j_n}|1\rangle)/\sqrt{2}$ are outputs. R_κ ($\kappa = 2, 3, \cdots, n$) are a series of phase transformations and H is the Hadamard transformation. The black dots present the control bits.

Correspondingly, the discrete QFT is defined as

$$|j\rangle \xrightarrow{QFT} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i j k/N}|k\rangle,$$

where $|k\rangle$ is a set of normative orthogonal basis. For simplicity, the transformation can be rewritten as the following form

$$\begin{aligned} |j_1\ \ |j_2 \cdots |j_n\rangle \xrightarrow{U_{QFT}} \\ (|0\rangle + e^{2\pi i 0.j_n}|1\rangle)(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle) \cdots \\ (|0\rangle + e^{2\pi i 0.j_2 \cdots j_n}|1\rangle)(|0\rangle + e^{2\pi i 0.j_1 j_2 \cdots j_n}|1\rangle)/2^{n/2}. \end{aligned}$$

where $0.j_1 j_2 \cdots j_n = j_1/2 + j_2/4 + \cdots + j_n/2^n$, j_l ($l = 1, 2, \cdots, n$) equal to 0 or 1. The effective circuit diagram of discrete QFT is shown in Fig. 1. The QFT is a unitary transformation, which is $R_\kappa = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^\kappa} \end{pmatrix}$.

Next, we investigate a detailed scenario for implementing the discrete QFT via coupled DQD molecules. There are n semiconductor coupled DQD molecules (GaAs) arranged in line, which is shown in Fig 2. Each molecule includes two electrons. According to the explicit analysis of Refs. [14, 15, 19], the charge states of each molecule (0,2), (1,1) and (2,0) can be transferred by sweeping the bias parameter Δ. For (n_1, n_2), n_1 (n_2) denotes the number of electrons in the upper (lower) quantum dots, and (0,2), (1,1) and (2,0) respectively correspond to $\Delta = E$, 0 and $-E$. E is the charging energy of each quantum dot. Δ can be controlled by gate-bias voltages of each molecule or by external electrical field. It is noted that

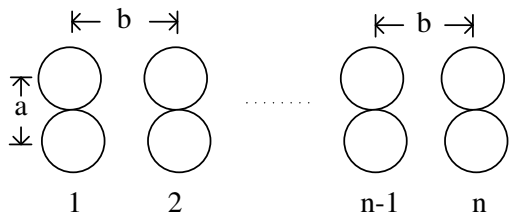

FIG. 2: Schematic diagram of coupled DQD molecules (the n identical molecules arranged in line). The hollow circle presents a quantum dot with one electron and two hollow circles construct a molecule. The distance between two dots of each molecule is a and the distance between two nearest-neighbor molecules is b.

the triplet state $|T\rangle$ will be in the charge state (1,1) and the singlet state $|S\rangle$ could be in (0,2), (1,1) or (2,0) if the initial charge state is in (1,1) during the process of sweeping bias parameter because of Pauli blockade.

In order to implement the discrete QFT, we prepare the n DQD molecules in spin states $|S\rangle = |0\rangle$ or $|T\rangle = |1\rangle$ randomly, and adjust $\Delta = 0$ (the charge states are all in (1,1)). Thus the total spin state is $|\psi\rangle = |j_1 j_2 \cdots j_n\rangle$, $j_l = |S\rangle$ or $|T\rangle$ ($l = 1, 2, \cdots, n$). The neighbor two DQD molecules are separated by an impenetrable barrier, so we only need consider the Coulomb interaction between two molecules. The detailed process can be described by the following n steps:

The first step: Firstly, we perform single-qubit gate transformations on DQD molecules 1 and 2, respectively,

$$|0\rangle_1 \to \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1), |1\rangle_1 \to \frac{1}{\sqrt{2}}(|0\rangle_1 - |1\rangle_1), \quad (1)$$

and

$$|0\rangle_2 \to |0\rangle_2, |1\rangle_2 \to e^{i\theta/2}|1\rangle_2, (\theta = \theta_{12} = \pi/2) \quad (2)$$

and a single-qubit operation on DQD molecules 1 as in Eq. (1), which can be achieved by Euler angle method or by U_Z rotations and U_{XZ} rotations in the XZ plane with finite singlet-triplet energy splitting [12]. Then we sweep the bias parameter Δ of molecules 1 and 2, the effect Hamiltonian is a Ising model, which can be expressed as [15, 19]

$$H = E_{12}\frac{1-\sigma_z^1}{2}\frac{1-\sigma_z^2}{2}, \quad (3)$$

where $E_{12} = \frac{1}{4\pi\epsilon}(\frac{2e^2}{b} - \frac{2e^2}{\sqrt{a^2+b^2}})$, ϵ is the dielectric constant of GaAs and σ_z is a Pauli operator. In this process, we adjust the interaction time t_{12}, let $E_{12}t_{12}/\hbar = (2\mu+1)\pi$, ($\mu \in \mathbb{N}$), thus we have the evolution

$$\begin{aligned} |00\rangle_{12} \to |00\rangle_{12}, |01\rangle_{12} \to |01\rangle_{12}, \\ |10\rangle_{12} \to |10\rangle_{12}, |11\rangle_{12} \to -|11\rangle_{12}. \end{aligned} \quad (4)$$

Then we perform the single-qubit gate transform of Eq. (1), and another single-qubit transform

$$|0\rangle_1 \to |0\rangle_1, |1\rangle_1 \to e^{-i\theta/2}|1\rangle_1, (\theta = \theta_{12} = \pi/2) \quad (5)$$

and the single-qubit transform of Eq. (1) again. We again weep Δ of molecules 1 and 2 to drive the interaction as in Eq. (3-4), perform the single-qubit transform of Eq. (1) and single-qubit transform as in Eq. (2) on molecule 1.

Secondly, we mainly consider the operations on molecules 1 and 3, the process is similar as above mentioned (from Eq. (2) to the end of above paragraph), in which $\theta = \theta_{13} = \pi/2^2 = \pi/4$, $E_{13} = \frac{1}{4\pi\epsilon}[\frac{2e^2}{2b} - \frac{2e^2}{\sqrt{a^2+(2b)^2}}]$. Similarly, we consider the molecules 1 and 4, molecules 1 and 5, up to molecules 1 and n one by one. In a word, the angle θ satisfies $\theta = \theta_{1\iota} = \pi/2^{\iota-1}$, the energy $E_{1\iota}$ will be $E_{1\iota} = \frac{1}{4\pi\epsilon}[\frac{2e^2}{(\iota-1)b} - \frac{2e^2}{\sqrt{a^2+(\iota-1)^2b^2}}]$, and the interaction time satisfies $E_{1\iota}t_{1\iota}/\hbar = (2\mu+1)\pi$, ($\iota = 2, 3, \cdots, n$). The state of molecule 1 becomes $m_{1\to n} = (|0\rangle + e^{2\pi i 0.j_l \cdots j_n}|1\rangle)/\sqrt{2}$.

The second step: Firstly, we still perform the single-qubit transformation on molecule 2 as in Eq. (1). Then considering the operations on molecules 2 and 3, 2 and 4, $\cdots$, 2 and n as the process of the first step. Here, $\theta = \theta_{2\iota} = \pi/2^{\iota-2}$, $E_{2\iota} = \frac{1}{4\pi\epsilon}[\frac{2e^2}{(\iota-2)b} - \frac{2e^2}{\sqrt{a^2+(\iota-2)^2b^2}}]$, and $E_{2\iota}t_{1\iota}/\hbar = (2\mu+1)\pi$, ($\iota = 3, 4, \cdots, n$). The state of molecule 2 becomes $m_{2\to n} = (|0\rangle + e^{2\pi i 0.j_2 \cdots j_n}|1\rangle)/\sqrt{2}$.

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The last step: We only need perform a single-qubit transformation on molecule n as in Eq. (1). The state of molecule n becomes $m_n = (|0\rangle + e^{2\pi i 0.j_n}|1\rangle)/\sqrt{2}$.

Finally, we read out the total result in reversed order, i.e., from molecule n to molecule 1. The result can be expressed as

$$\begin{aligned} |\Phi\rangle_{total} = & (|0\rangle + e^{2\pi i 0.j_n}|1\rangle)_n(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle)_{n-1} \\ & \cdots (|0\rangle + e^{2\pi i 0.j_1 j_2 \cdots j_n}|1\rangle)_1/2^{n/2}, \end{aligned} \quad (6)$$

which is the result of standard discrete QFT. By far, we have completed the task successfully. It is shown that, during the whole process, $\theta = \theta_{\gamma\iota} = \pi/2^{\iota-\gamma}$ and $E_{\gamma\iota} = \frac{1}{4\pi\epsilon}[\frac{2e^2}{(\iota-\gamma)b} - \frac{2e^2}{\sqrt{a^2+(\iota-\gamma)^2b^2}}]$, where $\gamma = 1, 2, \cdots, n-1$.

Initialization of the system is an important task for quantum computation. In the current scheme, the arbitrary initial state ($|S\rangle$ or $|T\rangle$) needs to be generated for an arbitrary QFT, however, for the practical QFT, the initial state must be a known one. In addition, the states $|S\rangle$ and $|T\rangle$ can be transformed each other. So we only need initialize the system to the state $|S\rangle$. As described in Ref. [14, 15], it can be realized by loading two electrons from a nearby Fermi sea into the ground state of a single quantum dot and then sweeping the bias parameter Δ from E to $-E$ by the rapid adiabatic passage to change state from (0,2) to (1,1). For the one-dimensional qubit array, we need at least two steps to initialize all qubit to the state $|S\rangle$ because the initialization can not be made on non-neighboring qubits simultaneously [14].

Next, we discuss the feasibility of the current scheme. Generally, the coherent time of electronic spin state will be affected by the hyperfine interaction between spins and nucleus. We choose the singlet and triplet spin states as qubits, which can weaken the negative effect efficiently. Charge fluctuations in environment, an important resource of decoherent

for charge qubits in semiconductor [20], which can lead to gate errors and dephase [3]. The unavoidable background charge noise, nuclear-spin-related noise and control electrical noise will affect the tunnel coupling between double quantum dots and the effective interaction strength between different molecules [15, 21]. All of above noise will result in an unwanted phase $\delta\phi$. Assume that the $\delta\phi$ has a Gaussian distribution $G(0,\sigma)$ with average value of zero and variance of σ [15, 22]. According to Ref. [15, 22], we can see that the fidelity of the Controlled-phase gate will be above 96% in this interaction model with $\delta\phi \simeq \pm 0.03\pi$. Moreover, this kind of noise can also be mended by adjusting the gate voltage of every molecule [23].

The spin coherent time can reach $1.2\mu s$ by using spin-echo technology [13]. In our scheme, the process mainly includes the controlled-R gate transformation. If the smallest Coulomb energy reduces to 0.1 percent of the largest Coulomb energy, *i.e.*, $\frac{E_{min}}{E_{max}} = \frac{\frac{1}{4\pi\varepsilon}(\frac{2e^2}{nb} - \frac{2e^2}{\sqrt{a^2+(nb)^2}})}{\frac{1}{4\pi\varepsilon}(\frac{2e^2}{b} - \frac{2e^2}{\sqrt{a^2+b^2}})} = 0.1\%$, the number of the DQD molecule will be $n \simeq 16$. Assume that $a = 5nm$ and $b = 12nm$, the total interaction time of the whole process will be $t_{total} \simeq 20ns$. The ratio is only 1.6% between the total interaction time and coherent time. Therefore we can complete the discrete QFT in the coherent time scale.

We should also pay attention to the process of interactions between two molecules. In order to avoid interaction noise coming from others quantum dot molecules, we can consider the electronic shield technology, *i.e.*, if one needs quantum dot molecules A and B to interact each other, others can be all shielded by electric shielding boxes put to earth. In this case, arbitrary two molecules interact each other with a different distance, and the interaction of them is governed by Eq. (3). A difficulty in experiment would be that the electric shielding box will slightly affect the interaction of the two DQD molecules with a certain extent. In addition, we separate any two neighbor molecules by impenetrable barriers, which can prevent the tunneling effect from two different molecules each other.

In conclusion, we present a scheme for implementing the discrete QFT via coupled semiconductor DQD molecules (GaAs). The main task is to realize controlled-R transformations $U(R)$. We can rewrite it by product form with some unitary transformations, i.e., $U_{12}(R) = U_2(\frac{\theta}{2})U_{cnot}U_2(-\frac{\theta}{2})U_{cnot}U_1(\frac{\theta}{2})$ or $U_{12}(R) = U_2(\frac{\theta}{2})U_2(H)U_{c-z}U_2(H)U_2(-\frac{\theta}{2})U_2(H)U_{c-z}U_2(H)U_1(\frac{\theta}{2})$. Every unitary transformation can be simply realized in our scheme. It is very important for the realization of universal quantum computation. Meanwhile, discrete QFT is a key ingredient for some quantum algorithms. Therefore, our scheme would be a useful step towards the realization of complex quantum algorithms in quantum dot system.

Acknowledgments

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